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Highlights

- We firstly assess the performance of participants in a two-stage Olympic process.
- We extend the relational model (Kao et al. 2008) to the VRS version.
- A heuristic search procedure is applied to the non-linear extended model.
- We prove that the efficiency for the entire two-stage Olympic process is unique.

Performance evaluation of participating nations at the 2012 London Summer Olympics by a two-stage data envelopment analysis

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Abstract:

This study measures the performance of participating nations at the Olympics, considering the quest for medals as a two-stage Olympic process. The first stage is characterized as athlete preparation (AP) and the second stage as athlete competition (AC). We extend the relational model from the *constant returns to scale* framework to the *variable returns to scale* version. The efficiency of each participating nation in the entire two-stage Olympic process is calculated as a product of the efficiencies of both stages, and a heuristic search is applied to the extended relational model. The efficiency of each stage can be obtained and directions for improving the performance of participating nations in the two-stage Olympic process can be identified. An empirical study of the 2012 London Summer Olympic Games reveals that the efficiency of the AP stage is higher than that of the AC stage for the majority of participants. In addition, a plot of the relationship between these three efficiencies shows that the efficiency of the entire two-stage Olympic process is more significantly related to that of the AC stage than that of the AP stage.

Keywords: data envelopment analysis; two-stage process; performance evaluation; heuristic search procedure.

1. Introduction

The Olympic Games is one of the most popular and most important sporting and cultural events in the world. All participating nations strive to obtain many medals to enhance their international prestige and presence on the world stage. A nation's success at the Olympics can be judged by various measures including number of gold medals, total number of medals, and either of these scaled by some demographic such as population. The Olympic Committee has never published an official ranking of participating nations (Lins et al., 2003). Consequently, many researchers have analyzed the performance of participating nations in the Olympics.

Data envelopment analysis (DEA) has been widely used to evaluate the relative efficiencies of participating nations in the Olympics. DEA is a popular non-parametric technique for measuring the relative efficiencies of peer decision-making units (DMUs). This technique is popular in efficiency evaluation because it makes no assumptions on the production function and imposes no subjective weights on multiple inputs and outputs.

Prior studies on DEA-based Olympics achievements evaluation can be classified into two categories. The first category is based on a constant input model in which the input for each nation is assumed to be a constant (Hai, 2007; Soares de Mello et al., 2008; 2009). These studies have goals similar to multicriteria-based researches (Saaty, 2008; Sitarz, 2012; 2013). The second category is based on classical DEA models in which inputs vary with nations and correspond to some social economic variables. For example, Lozano et al. (2002) considered two inputs (GNP and population) and three outputs (total numbers of gold, silver, and bronze medals) to measure the performance of participating nations in five Summer Olympic Games (1984–2000). Lins et al. (2003) considered the limited number of medals to be won and proposed a zero-sum game DEA model to analyze the performance of participating nations. Churilov and Flitman (2006) linked self-organizing maps to a DEA model to rank participating nations. To increase the validity of evaluation results, both Lozano et al. (2002) and Lins et al. (2003) applied the same set of assurance region (AR) constraints to all nations. Li et al. (2008) assumed that different nations impose different AR constraints and applied context-dependent AR DEA to measuring the performance of participating nations. Zhang et al. (2009) discussed underlying preferences in DEA and proposed DEA models with lexicographic preference to measure performance. Wu et al. (2009a) used a cross-efficiency DEA model to effectively rank participating nations and incorporated cluster analysis to effectively set frontier targets for inefficient DMUs. Considering competition among participating nations, Wu at al. (2009b) modified the game cross-efficiency model of Liang et al. (2008a) to assess the performance of participating nations in the Summer Olympics. Because numbers of medals are always integers, Wu et al. (2010) employed an integer-valued DEA model to measure the performance of each participating nation in the 2008 Beijing Olympics. Soares de Mello et al. (2012) proposed a non-radial DEA model to evaluate all participating nations in the 2008 Olympics, in which the input "population of each nation" is regarded as a nondiscretionary variable. Benicio et al. (2013) considered one input (the number of athletes) and three outputs (numbers of gold, silver, and bronze medals won) to measure the performance of nations in the 2010 Winter Olympics via an input-oriented, non-convex DEA model.

All of these studies treated each participating nation in the Olympics as a black box. That is, these studies ignore internal processes. When the internal processes of the DMU are considered, the efficiency score of the DMU can be assessed accurately and insights into the performance of the DMU can be obtained (Färe and Grosskopf, 2000). The two-stage DEA, which is the most common network DEA, opens the black box and has been applied to many areas, such as army recruitment (Charnes et al., 1986), education (Lovell et al., 1994), banking (Seiford and Zhu, 1999), physician care (Chilingerian and Sherman, 2004), information technology (Wang et al., 1997; Chen and Zhu, 2004), mutual funds (Premachandra et al., 2012), insurance companies (Kao and Hwang, 2008; Chen et al., 2009), and baseball (Sexton and Lewis, 2003). However, the two-stage DEA has not been used previously in research on the Olympic Games.

In this study, we employ the two-stage DEA to measure the performance of participating nations for three reasons. First, the two-stage DEA can reveal the hidden inefficiencies of participating nations in the Olympics as compared to conventional DEA models (Moreno and Lozano, 2012). Therefore, few nations may have perfectly efficient performance in the entire two-stage Olympic process. Second, based on the two-stage DEA, we can obtain the efficiency for each stage and identify inefficient stages for each nation. Third, exploring the black box of participating nations in the Olympics can provide decision-making guidance to improve their performance.

The two-stage Olympic process considered in this study is shown in Fig. 1. The first stage is characterized as the stage of athlete preparation (AP) which includes the cultivation, training and selection of participating athletes. The second stage is described as the stage of athlete competition (AC). In the AP stage, each nation uses two inputs (population and GDP per capita) to generate the one output (the number of participating athletes). Here, participating athletes are defined as the ones selected to participate in the Olympics. Regarding the choice of inputs in the AP stage, our model assumes that the greater the population a nation has, the more participating athletes can compete in the Olympics (Lins et al., 2003). It is better, however, to also consider the conditions for athletes' training and improvement of their capacities. There is no doubt that a wealthy nation can satisfy these conditions more easily. Our model assumes GDP per capita captures the most important element of the economic power of each participating nation. Thus, GDP per capita and population are two inputs of the AP stage. The output (the number of participating athletes) of the AP stage is referred to as the intermediate measure that links both stages as shown in Fig. 1. In the AC stage, the number of participating athletes is used as the input to produce three final outputs (the numbers of gold, silver, and bronze medals). The numbers of gold, silver, and bronze medals are selected as final outputs since Olympic achievement is measured with respect to medals won. Soares de Mello et al. (2012) and Benicio et al. (2013) used inputs and outputs similar to the AC stage of this paper.

<Insert Fig. 1 About Here>

This study extends the relational model (Kao and Hwang, 2008) or the centralized model (Liang et al., 2008b[1]) to measure performance of the two-stage Olympic process and individual stages for each nation. This study assumes output orientation, because it makes no sense to cut down the population and GDP per capita for inefficient nations as would be done if we used input orientation for Olympic evaluation. Also, this study assumes variable returns to scale (VRS), because population, GDP per capita, and the number of participating athletes of all nations vary greatly. As the relational model is extended under the VRS framework, a product of free variables appears in the model. Thus, the extended relational model cannot be transformed into a linear programming problem. But we can apply a heuristic search (Li et al., 2012) to calculating the global optimal solution for the extended relational model.

The rest of this study is as follows. In Section 2, several models are developed to measure the efficiencies of each nation and its two individual stages. In Section 3, the proposed models are applied to the 2012 London Summer Olympic Games and obtained results are discussed. In the last section, concluding remarks are given.

2. Two-stage methodology for analyzing the Olympics

Suppose there are n participating nations and each participating nation is denoted as a DMU. Each $DMU_j (j=1,2,\cdots n)$ uses two inputs $X_{ij} (i=1,2)$ to produce the intermediate measure $Z_{dj} (d=1)$ in the AP stage. Then, the intermediate measure is treated as an input to generate the final outputs $Y_{rj} (r=1,2,3)$ in the AC stage.

When treating the DMU as a black-box, inputs of the DMU are GDP per capita and population, and outputs are the numbers of gold, silver, and bronze medals that DMU_j wins. Thus the black-box efficiency of the participating nation DMU_0 under evaluation can be obtained by applying BCC model (Banker et al., 1984) as follows:

[1] In fact, the two models are equivalent. For details, see Cook et al. (2010).

$$\begin{aligned} &Min \quad \theta_0 = \frac{\sum_{i=1}^{2} v_i X_{i0} + u_0}{\sum_{r=1}^{3} u_r Y_{r0}} \\ &s.t. \quad \frac{\sum_{i=1}^{2} v_i X_{ij} + u_0}{\sum_{r=1}^{3} u_r Y_{rj}} \geq 1, \forall j \\ &\qquad u_1 - u_2 \geq \xi \\ &\qquad u_2 - u_3 \geq \xi \\ &\qquad u_1 - 2u_2 + u_3 \geq \xi \\ &\qquad v_i, u_r \geq 0, \forall i, r, \quad u_0, \textit{free}. \end{aligned} \tag{1}$$

where v_i and u_r are unknown non-negative weights attached to inputs and outputs, respectively. The three assurance regions (AR) $u_1 - u_2 \geq \xi$, $u_2 - u_3 \geq \xi$ and $u_1 - 2u_2 + u_3 \geq \xi$ indicate the relative importance among gold, silver, and bronze medals. For example, $u_1 - u_2 \geq \xi$ means that a gold medal is more important than a silver one, $u_2 - u_3 \geq \xi$ expresses the idea that a silver medal is more important than a bronze, and $u_1 - 2u_2 + u_3 \geq \xi$ indicates that the difference in importance between a gold medal and a silver medal should be greater than the difference between a silver medal and a bronze medal (Soares de Mello et al., 2009). The non-Archimedean infinitesimal ξ is imposed to avoid the special condition that the three medals are equally valued (Soares de Mello et al., 2009). Denote the optimal objective function value of model (1) as θ_0^{BCC*}, which is the reciprocal of the black-box efficiency for DMU_0. Thus, the black-box efficiency of DMU_0 is $e_0^{BCC*} = 1/\theta_0^{BCC*}$.

This study measures the performance of participating nations in a two-stage process. In prior two-stage DEA methods, two common methods are used for aggregating individual stages of a two-stage process: weight additive manner and multiplicative manner. The weight additive manner describes the efficiency of the entire two-stage process as a weighted average of efficiencies of individual stages. The weight attached to each stage reflects the relative importance of each stage. The multiplicative manner describes the efficiency of the entire two-stage process as a product of efficiencies of the two individual stages. In this study, we adopt the multiplicative manner because with this manner, if either of the individual stages has an efficiency score of zero, then the efficiency score of the entire two-stage process would be zero. That is, if a participating nation gains no medals in the AC stage, which means that the efficiency score of the AC stage is zero, then its Olympics achievement is zero regardless of its AP stage efficiency. If a nation does not participate in Olympics, then its AP stage efficiency would be zero and this case is not included in this study. Therefore, we combine the two stages in a multiplicative approach, namely $e_j = e_j^1 * e_j^2$. Moreover, output orientation is assumed in this paper, so we have $\theta_j = \theta_j^1 * \theta_j^2$ since $e_j = \frac{1}{\theta_j} = e_j^1 * e_j^2 = \frac{1}{\theta_j^1} * \frac{1}{\theta_j^2}$.

We extend the output-oriented relational model of Kao and Hwang (2008) from the constant returns to scale (CRS) framework to the VRS version as follows:

$$
\begin{aligned}
Min \quad & \theta_0 = \theta_0^1 \times \theta_0^2 = \frac{\sum_{i=1}^{2} v_i X_{i0} + u_0^1}{\eta_1^1 Z_{10}} \times \frac{\eta_1^2 Z_{10} + u_0^2}{\sum_{r=1}^{3} u_r Y_{r0}} \\
s.t. \quad & \frac{\sum_{i=1}^{2} v_i X_{ij} + u_0^1}{\eta_1^1 Z_{1j}} \geq 1, \forall j \\
& \frac{\eta_1^2 Z_{1j} + u_0^2}{\sum_{r=1}^{3} u_r Y_{rj}} \geq 1, \forall j \\
& u_1 - u_2 \geq \xi \\
& u_2 - u_3 \geq \xi \\
& u_1 - 2u_2 + u_3 \geq \xi \\
& v_i, \eta_1^1, \eta_1^2, u_r \geq 0. \forall i, r \quad u_0^1, u_0^2, \ free.
\end{aligned}
\tag{2}
$$

In model (2), the objective function expresses the efficiency of the entire two-stage Olympic process as a product of the two individual stages' efficiencies. The first two sets of constraints ensure the efficiency scores of the two stages do not exceed one and the last three constraints are ARs for output weights. Also, η_1^1 and η_1^2 are unknown weights attached to the intermediate measure for the AP stage and the AC stage, respectively.

Similar to Kao et al. (2008) and Liang et al. (2008b), this study assumes that the same weights are attached to the intermediate measure both for the AP and AC stages, i.e. $\eta_1^1 = \eta_1^2$. For instance, the manufacturer and retailer jointly determine the price in their contract to achieve maximum profit (Huang and Li, 2001). Therefore, we also assume that the ‘‘worth’’ or value of the intermediate variable is the same regardless of whether they are being viewed as inputs or outputs (Li et al., 2012). Another reason is that this assumption can represent the series relationship between the two stages (Chen et al., 2009). If we solve the two-stage DEA without this assumption, then our method is identical to independently employing the BCC model for each stage. As pointed out in a number of studies (Chen and Zhu, 2004; Liang et al., 2008b), applying two separate DEA analyses to the two individual stages cannot represent the conflict between the two stages. In fact, when the AP stage is inefficient, its efficiency can be improved by expanding the intermediate measure Z_{dj}, but such an action will deteriorate the AC stage efficiency since Z_{dj} is the input of that stage. Hence, this paper assumes $\eta_1^1 = \eta_1^2 = \eta_1$ and model (2) can then be converted into the following model:

$$
\begin{aligned}
& Min \quad \theta_0 = \theta_0^1 \times \theta_0^2 = \frac{\sum_{i=1}^{2} v_i X_{i0} + u_0^1}{\eta_1 Z_{10}} \times \frac{\eta_1 Z_{10} + u_0^2}{\sum_{r=1}^{3} u_r Y_{r0}} \\
& s.t. \quad \frac{\sum_{i=1}^{2} v_i X_{ij} + u_0^1}{\eta_1 Z_{1j}} \geq 1, \forall j \\
& \qquad \frac{\eta_1 Z_{1j} + u_0^2}{\sum_{r=1}^{3} u_r Y_{rj}} \geq 1, \forall j \\
& \qquad u_1 - u_2 \geq \xi \\
& \qquad u_2 - u_3 \geq \xi \\
& \qquad u_1 - 2u_2 + u_3 \geq \xi \\
& \qquad v_i, \eta_1, u_r \geq 0. \forall i, r \quad u_0^1, u_0^2, \ free.
\end{aligned} \tag{3}
$$

It is hard to transform model (3) into a linear model because it is a fractional programming problem and involves the product of free variables. To facilitate the linearization of this model, the heuristic search procedure of Li et al. (2012) is applied.

Consequently, we consider the following model:

$$\begin{aligned}
&Min \quad \theta_0^1 = \frac{\sum_{i=1}^{2} v_i X_{i0} + u_0^1}{\eta_1 Z_{10}} \\
&s.t. \qquad \frac{\sum_{i=1}^{2} v_i X_{ij} + u_0^1}{\eta_1 Z_{1j}} \geq 1, \forall j \\
&\qquad\quad \frac{\eta_1 Z_{1j} + u_0^2}{\sum_{r=1}^{3} u_r Y_{rj}} \geq 1, \forall j \\
&\qquad\quad u_1 - u_2 \geq \xi \\
&\qquad\quad u_2 - u_3 \geq \xi \\
&\qquad\quad u_1 - 2u_2 + u_3 \geq \xi \\
&\qquad\quad u_0^1 = u_0^2 = 0 \\
&\qquad\quad v_i, \eta_1, u_r \geq 0, \forall i, r.
\end{aligned} \tag{4}$$

Model (4) measures the CCR efficiency of the AP stage since $u_0^1 = u_0^2 = 0$. Denote the optimal objective function value of model (4) as θ_0^{1CCR*}, then we have $\theta_0^1 \leq \theta_0^{1\mathrm{CCR}*}$ since the feasible region of the model to measure θ_0^{1CCR*} is smaller than that of the model to measure θ_0^1 and the objective function is to seek the minimum value. Thus, the reciprocal of the AP stage efficiency would be determined in an interval of $\theta_0^1 \in [1, \theta_0^{1CCR*}]$.

Model (4) is a fractional programming problem but it can be converted into a linear model via the Charnes-Cooper (C-C) transformation (Charnes and Cooper, 1962). Then model (4) is equivalent to the following linear programming model.

$$\begin{aligned}
&Min \quad \theta_0^1 = \sum_{i=1}^{2} w_i X_{i0} \\
&s.t. \qquad \sum_{i=1}^{2} w_i X_{ij} - \pi_1 Z_{1j} \geq 0, \forall j \\
&\qquad\quad \pi_1 Z_{1j} - \sum_{r=1}^{3} \mu_r Y_{rj} \geq 0, \forall j \\
&\qquad\quad \pi_1 Z_{10} = 1 \\
&\qquad\quad \mu_1 - \mu_2 \geq \xi \\
&\qquad\quad \mu_2 - \mu_3 \geq \xi \\
&\qquad\quad \mu_1 - 2\mu_2 + \mu_3 \geq \xi \\
&\qquad\quad w_i, \pi_1, \mu_r \geq 0, \forall i, r.
\end{aligned} \tag{5}$$

Since $\theta_0^1 \in [1, \theta_0^{1CCR*}]$, θ_0^1 can be considered as a variable in measuring the overall efficiency score of the two-stage Olympic process. Therefore, model (3) can be rewritten as:

$$
\begin{aligned}
Min \quad & \theta_0 = \theta_0^1 \times \theta_0^2 = \theta_0^1 \times \frac{\eta_1 Z_{10} + u_0^2}{\sum_{r=1}^{3} u_r Y_{r0}} \\
s.t. \quad & \frac{\sum_{i=1}^{2} v_i X_{ij} + u_0^1}{\eta_1 Z_{1j}} \geq 1, \forall j \\
& \frac{\eta_1 Z_{1j} + u_0^2}{\sum_{r=1}^{3} u_r Y_{rj}} \geq 1, \forall j \\
& \frac{\sum_{i=1}^{2} v_i X_{i0} + u_0^1}{\eta_1 Z_{10}} = \theta_0^1 \\
& \theta_0^1 \in [1, \theta_0^{1CCR*}] \\
& u_1 - u_2 \geq \xi \\
& u_2 - u_3 \geq \xi \\
& u_1 - 2u_2 + u_3 \geq \xi \\
& v_i, \eta_1, u_r \geq 0, \forall i, r \quad u_0^1, u_0^2, \ free.
\end{aligned}
\tag{6}
$$

Model (6) can be converted into model (7) through the C-C transformation:

$$
\begin{aligned}
Min \quad & \theta_0 = \theta_0^1 \times (\pi_1 Z_{10} + \mu_0^2) \\
s.t. \quad & \sum_{i=1}^{2} w_i X_{ij} + \mu_0^1 - \pi_1 Z_{1j} \geq 0, \forall j \\
& \pi_1 Z_{1j} + \mu_0^2 - \sum_{r=1}^{3} \mu_r Y_{rj} \geq 0, \forall j \\
& \sum_{i=1}^{2} w_i X_{i0} + \mu_0^1 - \theta_0^1 \times \pi_1 Z_{10} = 0 \\
& \sum_{r=1}^{3} \mu_r Y_{r0} = 1 \\
& \theta_0^1 \in [1, \theta_0^{1CCR*}] \\
& \mu_1 - \mu_2 \geq \xi \\
& \mu_2 - \mu_3 \geq \xi \\
& \mu_1 - 2\mu_2 + \mu_3 \geq \xi \\
& w_i, \pi_1, \mu_r \geq 0, \forall i, r \quad \mu_0^1, \mu_0^2, \ free.
\end{aligned}
\tag{7}
$$

In order to calculate the optimal solution of model (7), we set $\theta_0^1 = \theta_0^{1CCR*} - k * \varepsilon$. Here ε is a step size for the heuristic search procedure, and $k = 0,1,2,\cdots,[k_{\max}]+1$ where $[k_{\max}]$ is the maximum integer of $(\theta_0^{1CCR*} - 1)/\varepsilon$. In solving model (7), we increase *k* from the initial value 0 to $[k_{\max}]+1$. Thus for each *k*, a given $\theta_0^1(k)$ is obtained and then model (7) can be solved by linear programming. Denote the optimal objective function value of model (7) corresponding to each k as $\theta_0^{v1}(k)$ and the corresponding efficiency of the entire two-stage Olympic process as $e_0^{v1}(k)$. Then, the global optimal solution of model (7) can be obtained as $\theta_0^{v1*} = \underset{k}{Min} \quad \theta_0^{v1}(k)$. Thus, the corresponding global optimal efficiency score of the entire two-stage Olympic process is $e_0^{v1*} = 1/\theta_0^{v1*}$ when the reciprocal of the AP stage efficiency θ_0^1 is considered as a variable.

When the two-stage Olympic process obtains its maximum efficiency as e_0^{v1*}, the maximum efficiency score of the AP stage is $\overline{e_0^1} = 1/\theta_0^1(k^*) = 1/(\theta_0^{1CCR*} - k^* \times \varepsilon)$, where $k^* = Max \quad \{k \mid \theta_0^{v1*} = \theta_0^{v1}(k)\}$. As a result, the corresponding minimum efficiency score for the AC stage is $\underline{e_0^2} = e_0^{v1*} / \overline{e_0^1}$ and we have $e_0^{v1*} = \underline{e_0^1} \times \overline{e_0^2}$.

Similarly, we can obtain the best possible CCR efficiency of the AC stage via the following model (8):

$$
\begin{aligned}
Min \quad & \theta_0^2 = \pi_1 Z_{10} \\
s.t. \quad & \textstyle\sum_{i=1}^{2} w_i X_{ij} - \pi_1 Z_{1j} \ge 0, \forall j \\
& \pi_1 Z_{1j} - \textstyle\sum_{r=1}^{3} \mu_r Y_{rj} \ge 0, \forall j \\
& \textstyle\sum_{r=1}^{3} \mu_r Y_{ro} = 1 \\
& \mu_1 - \mu_2 \ge \xi \\
& \mu_2 - \mu_3 \ge \xi \\
& \mu_1 - 2\mu_2 + \mu_3 \ge \xi \\
& w_i, \pi_1, \mu_r \ge 0, \forall i, r.
\end{aligned}
\tag{8}
$$

Denote the optimal objective function of model (8) as θ_0^{2CCR*}. Then the reciprocal of the AC stage efficiency can be determined in an interval of $[1, \theta_0^{2CCR*}]$, namely $\theta_0^2 \in [1, \theta_0^{2CCR*}]$. Thus model (3) can be rewritten as follows:

$$
\begin{aligned}
& Min \quad \theta_0 = \theta_0^1 \times \theta_0^2 = \frac{\sum_{i=1}^{2} v_i X_{i0} + u_0^1}{\eta_1 Z_{10}} \times \theta_0^2 \\
& s.t. \qquad \frac{\sum_{i=1}^{2} v_i X_{ij} + u_0^1}{\eta_1 Z_{1j}} \geq 1, \forall j \\
& \qquad\quad \frac{\eta_1 Z_{1j} + u_0^2}{\sum_{r=1}^{3} u_r Y_{rj}} \geq 1, \forall j \\
& \qquad\quad \frac{\eta_1 Z_{10} + u_0^2}{\sum_{r=1}^{3} u_r Y_{ro}} = \theta_0^2 \\
& \qquad\quad \theta_0^2 \in [1, \theta_0^{2CCR*}] \\
& \qquad\quad u_1 - u_2 \geq \xi \\
& \qquad\quad u_2 - u_3 \geq \xi \\
& \qquad\quad u_1 - 2u_2 + u_3 \geq \xi \\
& \qquad\quad v_i, \eta_1, u_r \geq 0, \forall i, r \quad u_0^1, u_0^2, \ free.
\end{aligned}
\tag{9}
$$

Since θ_0^2 is considered as a variable within the interval of $[1, \theta_{0\max}^2]$, we can convert model (9) into a linear programming model as follows via the C-C transformation.

$$
\begin{aligned}
& Min \quad \theta_0 = (\sum_{i=1}^{2} w_i X_{i0} + \mu_0^1) \times \theta_0^2 \\
& s.t. \qquad \sum_{i=1}^{2} w_i X_{ij} + \mu_0^1 - \pi_1 Z_{1j} \geq 0, \forall j \\
& \qquad\quad \pi_1 Z_{1j} + \mu_0^2 - \sum_{r=1}^{3} \mu_r Y_{rj} \geq 0, \forall j \\
& \qquad\quad \pi_1 Z_{10} + \mu_0^2 - \theta_0^2 \times \sum_{r=1}^{3} \mu_r Y_{r0} = 0 \\
& \qquad\quad \pi_1 Z_{10} = 1 \\
& \qquad\quad \theta_0^2 \in [1, \theta_0^{2CCR*}] \\
& \qquad\quad \mu_1 - \mu_2 \geq \xi \\
& \qquad\quad \mu_2 - \mu_3 \geq \xi \\
& \qquad\quad \mu_1 - 2\mu_2 + \mu_3 \geq \xi \\
& \qquad\quad w_i, \pi_1, \mu_r \geq 0, \forall i, r \quad \mu_0^1, \mu_0^2, \ free.
\end{aligned}
\tag{10}
$$

Let $\theta_0^2 = \theta_0^{2CCR*} - t\varepsilon, t = 0,1,2,\cdots[t_{\max}]+1$, where $[t_{\max}]$ is the maximum integer of $(\theta_0^{2CCR*} - 1)/\varepsilon$. For each *t*, we can obtain a given $\theta_0^2(t)$, and then model (10) can be solved by linear programming. Denote the optimal objective function value of model (10) corresponding to each *t* as $\theta_0^{v2}(t)$ and the corresponding efficiency of the entire two-stage Olympic process as $e_0^{v2}(t)$. Then, the global optimal solution of model (10) can be obtained as $\theta_0^{v2*} = \underset{t}{Min} \ \ \theta_0^{v2}(t)$. Therefore, the corresponding global optimal efficiency score of the entire two-stage Olympic process is $e_0^{v2*} = 1/\theta_0^{v2*}$ when treating the reciprocal of the AC stage efficiency as a variable.

The maximum efficiency score of the AC stage is $\overline{e_0^2} = 1/\theta_0^2(t^*) = 1/(\theta_0^{2CCR*} - t^*\varepsilon)$, where $t^* = Max \ \ \{t \mid \theta_0^{v2*} = \theta_0^{v2}(t)\}$. Then the corresponding minimum efficiency score of the AP stage is $\underline{e_0^1} = e_0^{v2*} / \overline{e_0^2}$ and we have $e_0^{v2*} = \underline{e_0^1} \times \overline{e_0^2}$.

To summarize, in the heuristic search procedure, when we consider the reciprocal of the AP stage efficiency as a variable, its maximum efficiency score $\overline{e_0^1}$ and the minimum efficiency score of the AC stage $\underline{e_0^2}$ can be obtained. Similarly, if we treat the reciprocal of the AC stage efficiency as a variable, and then the maximum efficiency score of this stage $\overline{e_0^2}$ and the minimum efficiency score of the AP stage $\underline{e_0^1}$ can also be obtained.

Theorem 1. For each DMU, $e_0^{v1*} = e_0^{v2*}$, where e_0^{v1*} and e_0^{v2*} are optimal efficiency scores of the two-stage Olympic process when the reciprocal of the AP stage efficiency θ_0^1 and the reciprocal of the AC stage efficiency θ_0^2 are considered as variables.

Proof: See Appendix A.

From Theorem 1, we have $e_0^{v1*} = e_0^{v2*}$ and we define e_0^* as the unique efficiency score for the two-stage Olympic process. Thus $e_0^{v1*} = e_0^{v2*} = e_0^*$. Note that if $\underline{e_0^1} = \overline{e_0^1}$, then $\overline{e_0^2} = \underline{e_0^2}$ because $e_0^{v1*} = \overline{e_0^1} \times \underline{e_0^2} = e_0^*$ and $e_0^{v2*} = \underline{e_0^1} \times \overline{e_0^2} = e_0^*$. In addition, if $\underline{e_0^1} = \overline{e_0^1}$ or $\overline{e_0^2} = \underline{e_0^2}$, the efficiency decomposition for the two stages would be uniquely determined. Define e_0^{1*} and e_0^{2*} as the unique efficiency scores for the AP stage and the AC stage, respectively.

3. Results and discussion

This section presents the results of applying our proposed DEA models to the data set of the 2012 London Summer Olympic Games. The data set consists of 85 participants who won at least one medal in the 2012 Summer Olympics. The data of inputs of two participants (Chinese Taipei and Hong Kong) are collected from the official website of each region's government while that of the other 83 participants are collected from the official website of World Bank (http://databank.worldbank.org/ddp/home.do). The intermediate measure and outputs are gathered from the official website of the Olympics (http://www.olympic.org/en/content/All-Olympic-results-since-1896/). Table 1 summarizes the descriptive statistics of the 85 participants. It shows that the input data vary greatly. For example, the populations of the 85 participants range from 67,675 to 1,344,130,000, and the standard deviation is 199,093,366.9246. The same phenomenon occurs for the other input (GDP per capita). Thus, the VRS assumption of the proposed models has been verified.

<Insert Table 1 About Here>

Now, we apply models (7) and (10) to measure efficiency scores of participants in the two-stage Olympic process. In order to solve these models, a heuristic search is introduced. Taking Britain (DMU 13) as an example, the reciprocal of its CCR efficiency score for the AP stage is $\theta_0^{1CCR*} = 2.0571$ according to model (5). Let $\theta_0^1 = \theta_0^{1CCR*} - k * \varepsilon$, $k = 0,1,2 \cdots [k_{\max}] + 1$, and set the step size to $\varepsilon = 0.01$[2]. The value of k changes from the initial value of 0 to the maximum of 106 because $[k_{\max}] + 1 = [(\theta_0^{1CCR*} - 1)/\varepsilon] + 1 = 106$. The reciprocal of the AP stage efficiency score that corresponds to each k can then be obtained. Table 2 shows the reciprocal of the AP stage efficiency and the optimal efficiency of the entire two-stage Olympic

[2] Here, we set $\varepsilon = 0.01$ just for illustration convenience. The smaller the value of ε we select, the more accurate the efficiency results are. However, if the value of ε is too small, we cannot obtain all optimal efficiencies corresponding to each *k* for the entire two-stage Olympic process since it exceeds the display capacity of Matlab.

process for Britain corresponding to each *k*. For example, the optimal efficiency scores of Britain are 0.4530 when $k = 66$ and 0.5427 when $k = 89$. The global optimal efficiency of Britain is $e_0^{v1*} = 0.6304$ at $k = 106$.

<Insert Table 2 About Here>

To further explain the relationship between the efficiency score and *k*, we display the efficiency trend curve in Fig. 2. It shows that the optimal efficiency score of Britain is monotonically increasing for $k \in [0,\ 106]$ and its global optimal efficiency score is obtained when $k = 106$. Thus, the maximum efficiency score of Britain is $e_0^{v1*} = 0.6304$ when the reciprocal of its AP stage efficiency is considered as a variable.

<Insert Fig. 2 About Here>

Table 3 shows the black-box efficiency score and the efficiency scores based on our proposed two-stage DEA models. The black-box efficiency e_0^{BCC} in the third column of Table 3 is calculated via model (1), which treats the DMU as a black box. Based on extended relational models and a step size of $\varepsilon = 0.00001$, the global optimal efficiency score of the two-stage Olympic process and the related efficiency scores of the individual stages are shown in columns 4 to 9 of Table 3. When considering the reciprocal of the AP stage efficiency as a variable, the columns 4 to 6 report the corresponding maximum efficiency score of this stage $\overline{e_0^1}$, the minimum efficiency score of the AC stage $\underline{e_0^2}$, and the global optimal efficiency of the entire two-stage Olympic process e_0^{v1*}. Similarly, considering the reciprocal of the AC stage efficiency as a variable, we can obtain the minimum efficiency score of the AP stage $\underline{e_0^1}$, the maximum efficiency score of the AC stage $\overline{e_0^2}$, and the global optimal efficiency score e_0^{v2*} as shown in the last three columns of Table 3.

<Insert Table 3 About Here>

As shown in Table 3, efficiency scores of all participants are in accord with Theorem 1. That is, the efficiency score of each participant is unique when either the reciprocal of the AP stage efficiency or that of the AC stage is considered a variable. For example, the global optimal efficiency of Afghanistan (DMU 1) is 0.0881 when the reciprocal of its AP stage efficiency is considered a variable. When the reciprocal of its AC stage efficiency is considered a variable, its global optimal efficiency is also 0.0881. Results in columns labeled $\overline{e_0^1}$, $\underline{e_0^1}$, $\overline{e_0^2}$, and $\underline{e_0^2}$ satisfy $\overline{e_0^1} = \underline{e_0^1}$ and $\overline{e_0^2} = \underline{e_0^2}$, indicating that the efficiency decomposition of the two individual stages of all DMUs is unique. For Afghanistan, the maximum and minimum efficiency scores of its AP stage are $\overline{e_0^1} = 0.2339$ and $\underline{e_0^1} = 0.2339$, and the maximum and minimum efficiency scores of its AC stage are $\overline{e_0^2} = 0.3767$ and $\underline{e_0^2} = 0.3767$. Thus, efficiency scores of the two individual stages of Afghanistan are uniquely determined.

Furthermore, it can be found in Table 3 that 17 participants are efficient when the DMU is treated as a black box, including Australia (DMU 5), Britain, and China (DMU 16). However, measuring the performance of each participant in a two-stage process yields only two efficient participants, namely China and Dominica (DMU 24). This result verifies the stronger discrimination power of the two-stage model compared to that of the conventional model.

Moreover, as shown in Table 3, the black-box efficiency of each participant according to model (1) is no lower than the two-stage process efficiency according to models (7) or (10). To intuitively describe this phenomenon, Fig. 3 plots black-box efficiencies versus two-stage process efficiencies of all participants. The diagonal in Fig. 3 shows that the black-box efficiency is equal to the two-stage process efficiency. The horizontal axis of Fig. 3 represents the black-box efficiency, and the vertical axis represents the two-stage process efficiency. All participants are distributed below the diagonal. For instance, there are 17 efficient participants based on the BCC model (1), but 15 of them are evaluated as inefficient based on the two-stage DEA models (7) or (10). This is due to the fact that the two-stage DEA model can identify more sources of inefficiency than the conventional BCC model.

<Insert Fig. 3 About Here>

It is interesting to note that the two-stage Olympic process is efficient if and only if its two individual stages are efficient as shown in Table 3. For example, Bahamas (DMU 7) is efficient in the AP stage but is inefficient in the AC stage, thus, Bahamas has an imperfect efficiency score over the entire two-stage process. As for China, it is efficient in its two individual stages and in the entire two-stage Olympic process. Thus, participants such as Bahamas can become efficient in their entire two-stage Olympic process by improving the performance in their inefficient stages.

In Table 3, the efficiency score of the AP stage is generally greater than that of the AC stage for the majority of participants, which indicates that they have poor performance in the competition stage. To illustrate this point, 18 participants are efficient in the AP stage, including Australia, Bahamas, China, and Dominica, while only 3 participants, China, Dominica, and the USA (DMU 83) are efficient in the AC stage. The efficient state of these participants in the AP or AC stage may be due to a property of the conventional VRS model that DMUs with the smallest value of one input or the biggest value of one output are always efficient (Chen et al., 2002). For instance, in the AP stage, Dominica (smallest population), Ethiopia (smallest GDP per capita) and Britain (biggest number of athletes) are evaluated as efficient because of this property. In the AC stage, the USA (biggest numbers of gold and silver medals) is another example of this case. However, Belarus (DMU 9) with the biggest number of bronze medals is inefficient in the AC stage. This is due to the fact that the proposed models in this paper include weight restrictions on output multipliers while the conventional VRS model does not. Thus, the afore-mentioned property of the conventional VRS model may be invalid in some cases of our models.

Since the output of the AP stage is the input of the AC stage, the data of the AP stage are related to the data in the AC stage. Thus, the Wilcoxon signed-rank test is performed to investigate the relationship among the three efficiencies.

<Insert Table 4 About Here>

The results of the Wilcoxon signed-rank test in Table 4 suggest that the null hypotheses of any two of the three efficiencies are rejected at the 5% significance level. For example, the Wilcoxon signed-rank test for the null hypothesis of $e_0^* = e_0^{1*}$ returns the result of $h = 1$. Thus we reject the hypothesis of $e_0^* = e_0^{1*}$ since $h = 1$ indicates a rejection of the null hypothesis at the 5% significance level. That is, the efficiency of the entire two-stage process is different from that of the AP stage. Results of the Wilcoxon signed-rank test suggest that the performance evaluation of participants may be different based on different efficiencies. To enhance the relationship among these three efficiencies, the correlation coefficients between each pair of efficiencies are obtained as listed in Table 5.

<Insert Table 5 About Here>

Table 5 shows that the correlation coefficient of 0.8521 is statistically significant between efficiencies of the entire two-stage process and the AC stage. This result indicates that a participant with excellent performance in the AC stage might possibly have excellent performance in the entire two-stage Olympic process, because the achievements of participants are usually determined by the performance of their athletes in the competition stage. As for the efficiency of the AP stage, its correlation coefficient in relation to the efficiency of the entire two-stage process is 0.7235, which is also significant. However, the correlation coefficient between efficiencies of both stages is 0.3702, the lowest value of correlation. In such cases, excellent performance in the AP stage does not imply excellent performance in the AC stage. This may due to the fact that the output of the AP stage is the input for the AC stage. A possible incoherence in the efficiency may arise that the more efficient a participant is in its AP stage, the less efficient it is in its AC stage. The participant can be efficient in the entire two-stage process only when an appropriate number of participating athletes is selected and the number of athletes is in accord with the number of medals won, so one of advantages of the proposed two-stage model is that it can detect the source of poor performance for each participant.

Based on the results of the correlation analysis, strategy implications are obtained for two different situations. The first situation is that only one of the two stages is inefficient, and the second is that both of the two stages are inefficient and have similar efficiencies. For the first situation, an effective strategy of improving the performance of the two-stage Olympic process is to improve the efficiency of the inefficient stage. This is due to facts that the efficiency of the two-stage process is highly correlated to that of the AP and AC stages, and the efficiency of the AP stage is only slightly correlated to that of the AC stage. For example, if the AC stage is inefficient, then the output-oriented model will suggest that the DMU improve participating athletes' competitive ability to gain more medals in the Olympics and keep the number of athletes unchanged. If the AP stage is inefficient, then the output-oriented model will suggest that the DMU increase the number of athletes selected to join the Olympics, although increasing the number of athletes may decrease the efficiency of AC stage. However, the correlation analysis shows the AC stage efficiency is only slightly correlated to the AP stage efficiency. Thus, the improvement in the AP stage may have little effect on the AC stage efficiency. For the second situation, the effective strategy for improving the performance of the two-stage Olympic process is giving priority to improving the AC stage efficiency. This is due to the fact that correlation coefficient between efficiencies of the entire two-stage process and the AC stage is relatively higher than that between the entire two-stage process and the AP stage. Thus, the participant should improve athletes' competitive ability firstly to improve the performance of the two-stage Olympic process in this situation.

Since efficiency scores of both stages are uniquely determined, we can gain further insights into the performance of participants. Directions for improving the performance of participants in the entire two-stage Olympic process can also be identified. Thus, utilizing efficiencies of both stages, an efficiency matrix is established as shown in Fig. 4.

<Insert Fig. 4 About Here>

In Fig. 4, the horizontal and vertical axes of the efficiency matrix represent the AP stage efficiency and AC stage efficiency, respectively, and each participant is located in the matrix. The average efficiency of the AP stage of the 85 participants is 0.5629, and that of the AC stage is 0.3712. Using both average efficiencies, we can divide the efficiency matrix into four quadrants. According to the location of the participant in the four quadrants, policy and strategic implications for improving performance can be proposed.

Located in the second quadrant, Iran (DMU 40) has a high efficiency of 0.9138 in the AC stage but a low efficiency of 0.1791 in the AP stage. Iran can improve its performance in the entire two-stage Olympic process by exerting considerable effort in the AP stage. Thus, increasing the number of participating athletes may be an appropriate strategy. However, increasing the number of participating athletes may result in deterioration in the AC stage efficiency since the number of participating athletes is an input for that stage. In other words, it is unreasonable for a participant to brashly expand the number of participating athletes unless the participating athletes can obtain a corresponding number of medals. Thus, cultivating and selecting more potential elite athletes may be a suitable strategy. Naturally, this strategy requires enough resources (financial, material and manpower) for athlete training and improvement of their capacities. Therefore, improving the economic strength of the participant is a long-term strategy. In contrast, participants in the fourth quadrant exhibit high AP stage efficiency but low AC stage efficiency. The policy implication for these participants, such as Poland (DMU 61) and Croatia (DMU 19), is giving priority to improving their performance in the AC stage. Thus, these participants should put emphasis on improving their athletes' competitive ability. Another consideration is that a relatively low AC stage efficiency and high AP stage efficiency may be due to a low standard of selecting participating athletes. In such a situation, an excess number of athletes acquire the chance of participating in the Olympics. Thus, establishing an appropriate standard of selecting participating athletes may be an alternative strategy.

Participants in the third quadrant have low efficiencies in both stages. For example, the AP stage and AC stage efficiencies of Saudi Arabia (DMU 67) are 0.0601 and 0.1751, respectively, both of which are lower than the averages. Participants in the third quadrant can be classified into three kinds, and then corresponding strategic implications can be proposed. The first kind is participants whose AP stage efficiency is significantly higher than the AC stage efficiency. The strategic implication for these participants could be giving priority to improving athletes' competitive ability. The second kind is the participants whose AP stage efficiency is significantly lower than the AC stage efficiency. For these, the strategic implication is to appropriately increasing the number of participating athletes. The third kind of participants is those whose AP stage efficiency is almost equal to the AC stage efficiency. Then, based on the strategic implications of the correlation analysis, the implication for this kind of participant is to improve the AC stage efficiency. Thus, improving participating athletes' competitive ability should be a suitable strategy. Participants in the first quadrant have relatively higher efficiency scores in both stages compared to other participants. The two efficient participants in the entire two-stage Olympic process, China and Dominica are located in the first quadrant. Russia (DMU 66) is also in the first quadrant, with an efficient AP stage and a high efficiency of 0.8858 in the AC stage. Participants such as Russia with only one inefficient stage can adopt strategy according to the effective strategy of the correlation analysis. Other participants in the first quadrant can be classified into the same three kinds of participants as in the third quadrant. Thus, the same strategic implications can be adopted for corresponding kinds of participants.

4. Conclusions

The performance evaluation of participating nations in the Olympics Games is a classical two-stage process. However, no previous study has used the two-stage DEA to measure the performance of participating nations in the Olympics. The current study evaluates such performance in a two-stage process. The individual stages are characterized as the AP (athlete preparation, including athlete cultivation, training, and selection) and AC (athlete competition) stages. Variable return to scale and output orientation are assumed. The weights of output multipliers are restricted to ensure the validity of the results. That is, a gold medal is more important than a silver medal and the latter is more important than a bronze medal, and the difference in importance between a gold medal and a silver medal should be greater than the difference between a silver medal and a bronze medal. A heuristic search is used to estimate the efficiency score of the entire two-stage Olympic process because of nonlinearity in the extended relational model. By applying the proposed models to the real data set of 85 participants in the 2012 London Summer Olympics, their efficiency scores for the entire two-stage Olympic process and the two stages can be obtained. The efficiency of the AP stage is usually higher than that of the AC stage. Moreover, the correlation between efficiencies of the entire two-stage Olympic process and the AC stage is more significant than that between the entire process and the AP stage.

In this paper, the output multipliers of all participating nations have the same weight restrictions. However, different nations may value gold, silver, and bronze medals differently. Thus, how to impose different weight restrictions on output multipliers for different nations in the two-stage DEA model is a possible direction of future studies.

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Appendix A

Theorem 1. For each DMU, $e_0^{v1*} = e_0^{v2*}$, where e_0^{v1*} and e_0^{v2*} are optimal efficiency scores of the two-stage Olympic process when the reciprocal of the AP stage efficiency θ_0^1 and the reciprocal of the AC stage efficiency θ_0^2 are considered as variables.

Proof: When either the reciprocal of the AP stage efficiency θ_0^1 or the reciprocal of the AC stage efficiency θ_0^2 is considered as a variable, the optimal efficiency score for the same two-stage Olympic process is unique (Li et al., 2012). Thus, we have $e_0^{v1*} = e_0^{v2*}$.

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Figures

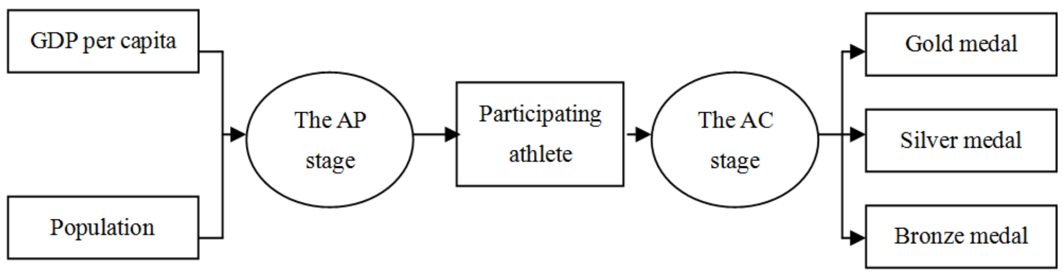

Fig. 1. Two-stage Olympic process.

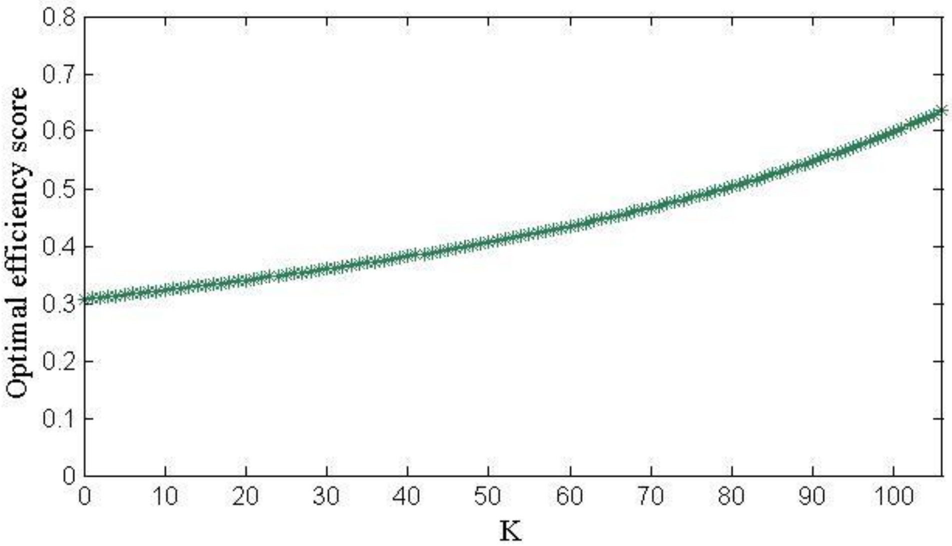

Fig. 2. Efficiency trend for Britain (DMU 13) based on model (7).

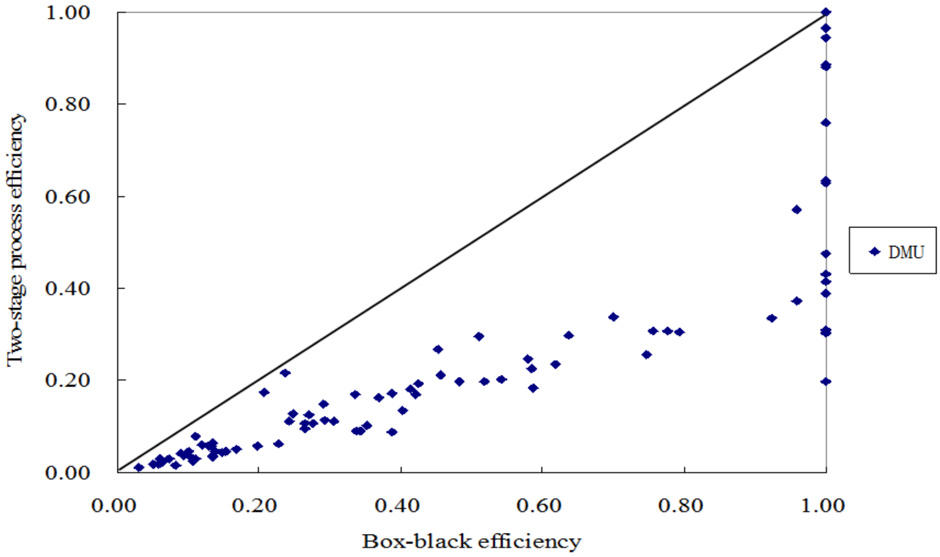

Fig. 3. Two-stage process efficiency vs. black-box efficiency.

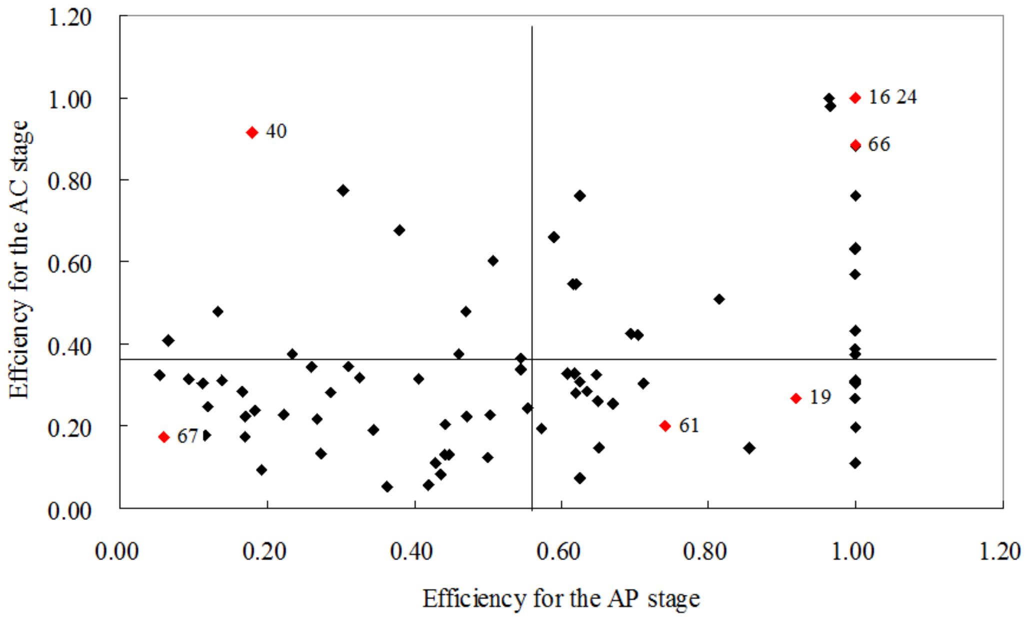

Fig. 4. AP stage efficiency vs. AC stage efficiency.

Tables

Table 1 Descriptive statistics of raw data.

Stat.	Input		Intermediate measure	Output		
	GDP per capita	Population	Participating athlete	Gold medal	Silver medal	Bronze medal
Max	98080.9129	1344130000	556	46	29	32
Min	356.9673	67675	2	0	0	0
Average	21419.3783	63376957.2353	114.1882	3.5529	3.5765	4.1882
Std.Dev	22096.46	199093366.9246	123.8155	7.6306	5.9809	6.0326

Table 2 Efficiency trend of Britain (DMU 13) based on model (7).

K	$\theta_0^1(k) = \theta_0^{1CCR*} - k*0.01$	$e_0^{v1}(k)$
0-10	2.0517-1.9517	0.3073-0.3230
11-20	1.9417-1.8517	0.3247-0.3405
21-30	1.8417-1.7517	0.3423-0.3599
31-40	1.7417-1.6517	0.3620-0.3817
41-50	1.6417-1.5517	0.3840-0.4063
51-60	1.5417-1.4517	0.4089-0.4343
61-70	1.4417-1.3517	0.4373-0.4664
71-80	1.3417-1.2517	0.4699-0.5037
81-90	1.2417-1.1517	0.5077-0.5474
91-100	1.1417-1.0517	0.5522-0.5995
101-106	1.0417-0.9917	0.6052-0.6304 (global optimal efficiency)

Table 3 Efficiency results based on $\varepsilon = 0.00001$.

DMU	Participant	e_0^{BCC*}	Stage 1 as a variable			Stage 2 as a variable		
			$\overline{e_0^1}$	$\underline{e_0^2}$	e_0^{v1*}	$\underline{e_0^1}$	$\overline{e_0^2}$	e_0^{v2*}
1	Afghanistan	0.3873	0.2339	0.3767	0.0881	0.2339	0.3767	0.0881
2	Algeria	0.1336	0.1707	0.2235	0.0382	0.1707	0.2235	0.0382
3	Argentina	0.1188	0.4993	0.1229	0.0613	0.4993	0.1229	0.0613
4	Armenia	0.5100	0.6941	0.4273	0.2966	0.6941	0.4273	0.2966
5	Australia	1.0000	1.0000	0.3886	0.3886	1.0000	0.3886	0.3886
6	Azerbaijan	0.6190	0.3041	0.7744	0.2355	0.3041	0.7744	0.2355
7	Bahamas	0.7563	1.0000	0.3073	0.3073	1.0000	0.3073	0.3073
8	Bahrain	0.1479	0.1838	0.2391	0.0440	0.1838	0.2391	0.0440
9	Belarus	0.7760	1.0000	0.3069	0.3069	1.0000	0.3069	0.3069
10	Belgium	0.1379	0.4294	0.1089	0.0468	0.4294	0.1089	0.0468
11	Botswana	0.1059	0.0650	0.4104	0.0267	0.0650	0.4104	0.0267
12	Brazil	0.2075	0.6190	0.2830	0.1752	0.6190	0.2830	0.1752
13	Britain	1.0000	1.0000	0.6304	0.6304	1.0000	0.6304	0.6304
14	Bulgaria	0.1282	0.4409	0.1306	0.0576	0.4409	0.1306	0.0576
15	Canada	0.4142	0.6351	0.2862	0.1817	0.6351	0.2862	0.1817
16	China	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	Chinese Taipei	0.0589	0.1169	0.1791	0.0209	0.1169	0.1791	0.0209
18	Colombia	0.2472	0.4062	0.3155	0.1282	0.4062	0.3155	0.1282
19	Croatia	0.5785	0.9196	0.2692	0.2476	0.9196	0.2692	0.2476
20	Cuba	0.9228	0.6150	0.5474	0.3367	0.6150	0.5474	0.3367
21	Cyprus	0.1674	0.2225	0.2272	0.0506	0.2225	0.2272	0.0506
22	Czech Republic	0.5415	0.6182	0.3269	0.2021	0.6182	0.3269	0.2021
23	Denmark	0.5860	0.5447	0.3375	0.1839	0.5447	0.3375	0.1839
24	Dominica	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	Egypt	0.0998	0.6260	0.0738	0.0462	0.6260	0.0738	0.0462
26	Estonia	0.2932	0.5034	0.2281	0.1149	0.5034	0.2281	0.1149
27	Ethiopia	1.0000	1.0000	0.7606	0.7606	1.0000	0.7606	0.7606
28	Finland	0.1981	0.2681	0.2176	0.0583	0.2681	0.2176	0.0583
29	France	0.5185	0.6079	0.3269	0.1987	0.6079	0.3269	0.1987
30	Gabon	0.1323	0.4475	0.1303	0.0583	0.4475	0.1303	0.0583
31	Georgia	1.0000	0.6258	0.7606	0.4760	0.6258	0.7606	0.4760
32	Germany	0.6373	0.7032	0.4231	0.2975	0.7032	0.4231	0.2975
33	Greece	0.0929	0.4367	0.0839	0.0367	0.4367	0.0839	0.0367
34	Grenada	0.9589	1.0000	0.5710	0.5710	1.0000	0.5710	0.5710
35	Guatemala	0.1108	0.1699	0.1751	0.0297	0.1699	0.1751	0.0297
36	Hong Kong	0.0584	0.1926	0.0932	0.0179	0.1926	0.0932	0.0179
37	Hungary	1.0000	0.8143	0.5081	0.4138	0.8143	0.5081	0.4138
38	India	0.2368	0.7112	0.3049	0.2168	0.7112	0.3049	0.2168
39	Indonesia	0.0593	0.0928	0.3142	0.0292	0.0928	0.3142	0.0292

40	Iran	0.3684	0.1791	0.9138	0.1636	0.1791	0.9138	0.1636
41	Ireland	0.3523	0.3265	0.3174	0.1036	0.3265	0.3174	0.1036
42	Italy	0.4522	1.0000	0.2692	0.2692	1.0000	0.2692	0.2692
43	Jamaica	1.0000	0.9651	0.9789	0.9447	0.9651	0.9789	0.9447
44	Japan	0.4823	0.5450	0.3654	0.1991	0.5450	0.3654	0.1991
45	Kazakhstan	0.7931	0.5075	0.6028	0.3060	0.5075	0.6028	0.3060
46	Kenya	1.0000	1.0000	0.8816	0.8816	1.0000	0.8816	0.8816
47	Korea People	1.0000	0.5900	0.6589	0.3888	0.5900	0.6589	0.3888
48	Korea Republic.	0.6999	0.6192	0.5458	0.3379	0.6192	0.5458	0.3379
49	Kuwait	0.0824	0.0538	0.3235	0.0174	0.0538	0.3235	0.0174
50	Latvia	0.3049	0.5737	0.1949	0.1118	0.5737	0.1949	0.1118
51	Lithuania	0.4559	0.6484	0.3265	0.2117	0.6484	0.3265	0.2117
52	Malaysia	0.0728	0.1203	0.2466	0.0297	0.1203	0.2466	0.0297
53	Mexico	0.1098	0.2861	0.2809	0.0804	0.2861	0.2809	0.0804
54	Moldova	1.0000	1.0000	0.3038	0.3038	1.0000	0.3038	0.3038
55	Mongolia	1.0000	1.0000	0.6334	0.6334	1.0000	0.6334	0.6334
56	Montenegro	0.2416	1.0000	0.1113	0.1113	1.0000	0.1113	0.1113
57	Morocco	0.0639	0.4185	0.0579	0.0242	0.4185	0.0579	0.0242
58	Netherlands	0.7475	0.3794	0.6779	0.2572	0.3794	0.6779	0.2572
59	New Zealand	1.0000	1.0000	0.3107	0.3107	1.0000	0.3107	0.3107
60	Norway	0.3427	0.2603	0.3455	0.0899	0.2603	0.3455	0.0899
61	Poland	0.2900	0.7410	0.2008	0.1488	0.7410	0.2008	0.1488
62	Portugal	0.0504	0.3640	0.0526	0.0191	0.3640	0.0526	0.0191
63	Puerto Rico	0.1518	0.1674	0.2849	0.0477	0.1674	0.2849	0.0477
64	Qatar	0.2265	0.1331	0.4782	0.0636	0.1331	0.4782	0.0636
65	Romania	0.3874	0.4609	0.3743	0.1725	0.4609	0.3743	0.1725
66	Russia	1.0000	1.0000	0.8858	0.8858	1.0000	0.8858	0.8858
67	Saudi Arabia	0.0290	0.0601	0.1751	0.0105	0.0601	0.1751	0.0105
68	Serbia	0.2698	0.8549	0.1464	0.1252	0.8549	0.1464	0.1252
69	Singapore	0.1342	0.1114	0.3038	0.0338	0.1114	0.3038	0.0338
70	Slovakia	0.2639	0.3103	0.3447	0.1070	0.3103	0.3447	0.1070
71	Slovenia	0.4200	0.6711	0.2539	0.1704	0.6711	0.2539	0.1704
72	South Africa	0.2754	0.4726	0.2255	0.1066	0.4726	0.2255	0.1066
73	Spain	0.3356	0.6489	0.2621	0.1701	0.6489	0.2621	0.1701
74	Sweden	0.4019	0.5541	0.2441	0.1352	0.5541	0.2441	0.1352
75	Switzerland	0.3375	0.4425	0.2041	0.0903	0.4425	0.2041	0.0903
76	Tajikistan	1.0000	1.0000	0.1978	0.1978	1.0000	0.1978	0.1978
77	Thailand	0.0903	0.1383	0.3112	0.0430	0.1383	0.3112	0.0430
78	Trinidad & Tobago	0.5843	0.4707	0.4802	0.2260	0.4707	0.4802	0.2260
79	Tunisia	0.2643	0.6505	0.1475	0.0960	0.6505	0.1475	0.0960
80	Turkey	0.1349	0.3452	0.1909	0.0659	0.3452	0.1909	0.0659
81	Uganda	1.0000	1.0000	0.4320	0.4320	1.0000	0.4320	0.4320
82	Ukraine	0.9589	1.0000	0.3737	0.3737	1.0000	0.3737	0.3737

83	USA	1.0000	0.9640	1.0000	0.9640	0.9640	1.0000	0.9640
84	Uzbekistan	0.4237	0.6255	0.3097	0.1938	0.6255	0.3097	0.1938
85	Venezuela	0.1003	0.2725	0.1354	0.0369	0.2725	0.1354	0.0369

Table 4 Wilcoxon signed-rank test results for three efficiencies.

$H_0: e_0^* = e_0^{1*}; H_1: e_0^* \neq e_0^{1*}$	$h=1$ (5%)
$H_0: e_0^* = e_0^{2*}; H_1: e_0^* \neq e_0^{2*}$	$h=1$ (5%)
$H_0: e_0^{1*} = e_0^{2*}; H_1: e_0^{1*} \neq e_0^{2*}$	$h=1$ (5%)

Notes: (1) The formula of $h=1$ indicates a rejection of the null hypothesis H_0. (2) The figures in parentheses stand for the significance level

Table 5 Correlation coefficients between three efficiencies.

	e_0^*	e_0^{1*}	e_0^{2*}
e_0^*	1	0.7235	0.8521
e_0^{1*}		1	0.3702
e_0^{2*}			1