Corrigendum to "Equilibrium decisions for an innovation crowdsourcing platform"

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This note corrects an error in the proof of Proposition 7 of Cheng et al. (2019, *Transportation Research Part E* 125, 241–260). The error is located in Eq. (A.21) of the proof, where the first mean value theorem for definite integrals is applied although its prerequisite fails to hold. We provide a new, fully rigorous proof of a stronger result: the single-prize contest is the unique globally optimal award mechanism for the sponsor. Consequently, the conclusion of Proposition 7 and all results built upon it—including Corollary 2, Propositions 8–12, Corollary 3, and the statements in the abstract—remain valid and unchanged.

1 The error

In the proof of Proposition 7 (Appendix A, p. 257), the difference between the sponsor's expected payoffs under the original N-prize mechanism and the constructed $(N-1)$-prize mechanism is written as (Eq. (A.21))

$$\Pi_M - \tilde{\Pi}_M \;=\; kn \int_0^1 \psi(a_i)\,\varphi(a_i)\,da_i,$$

where $\psi(a_i) = C(a_i) - \tilde{C}(a_i)$ and $\varphi(a_i) = a_i^{n-1}\big/\big(\sqrt{A_1(a_i)+C(a_i)} + \sqrt{A_1(a_i)+\tilde{C}(a_i)}\big) > 0$. The proof then invokes the first mean value theorem for definite integrals to claim the existence of $\xi \in (0,1)$ such that $\int_0^1 \psi(a_i)\varphi(a_i)\,da_i = \varphi(\xi)\int_0^1 \psi(a_i)\,da_i$, and infers the sign of $\Pi_M - \tilde{\Pi}_M$ from $\varphi(\xi) > 0$ together with $\int_0^1 \psi\,da_i < 0$.

This application of the first mean value theorem is invalid. The theorem requires the function that remains under the integral sign—here ψ—to be of constant sign on $[0,1]$. However, combining the definitions of $C(a_i)$ and $\tilde{C}(a_i)$ in the paper gives

$$\psi(a) \;=\; \frac{(n-1)!\,R_N}{(N-1)!\,(n-N)!}\int_0^a t^{\,n-N}(1-t)^{N-2}\big[(n-N)-nt\big]\,dt, \tag{1}$$

and the factor $(n-N)-nt$ in the integrand is positive for $t < (n-N)/n$ and negative for $t > (n-N)/n$. Hence, whenever $3 \le N \le n-1$, $\psi(a) > 0$ for sufficiently small $a > 0$, while direct evaluation with the beta function yields

$$\psi(1) \;=\; -\,\frac{(n-1)!\,R_N}{(N-1)!\,(n-N)!}\cdot\frac{(N-2)!\,(n-N)!}{(n-1)!} \;<\; 0. \tag{2}$$

Thus ψ changes sign on $[0,1]$, the prerequisite of the mean value theorem does not hold, and the proof of Proposition 7 as printed is incorrect. The error was identified by a member of our group during a careful examination of the paper. We regret the oversight.

2 The corrected result

The following theorem replaces Proposition 7. It is stronger than the original statement: rather than comparing an N-prize mechanism with a particular $(N-1)$-prize mechanism, it establishes

directly that the single-prize contest uniquely maximizes the sponsor's expected payoff among all reasonable award mechanisms with the same total award. All notation is as in the original paper.

Theorem 1. *Consider the model of Section 4 of the original paper, with $n \geq 3$ participants whose abilities are i.i.d. uniform on $[0,1]$. Among all reasonable award mechanisms with N prizes, $1 \leq N \leq n$, satisfying $R_1 \geq R_2 \geq \cdots \geq R_N \geq 0$ and $\sum_{j=1}^{N} R_j = R$, the sponsor's expected payoff is uniquely maximized by the single-prize mechanism, i.e., $N = 1$ and $R_1 = R$.*

Proof. The proof proceeds in five steps.

Step 1: Equilibrium effort. Under a reasonable award mechanism (Definition 1 of the paper), let $b(\cdot)$ be the symmetric, increasing equilibrium effort strategy. A participant with ability a finishes in j-th place with probability $p_j(a) = \binom{n-1}{j-1}(1-a)^{j-1}a^{n-j}$, and her expected payoff is $u(a) = \sum_{j=1}^{N} R_j p_j(a) - b^2(a)/a$. The first-order condition, exactly as in the proofs of Propositions 2, 4, and 6 of the paper, gives $(b^2)'(a) = a\sum_{j=1}^{N} R_j\, p_j'(a)$. Let $Q_j(t) = \sum_{i=1}^{j} p_i(t)$, $j = 0, 1, \ldots, n$, denote the probability that the participant finishes in one of the top j places—equivalently, that she wins one of the prizes $R_1, \ldots, R_j$—with $Q_0 \equiv 0$ and $Q_n \equiv 1$. For $1 \leq j \leq n-1$, this is also the probability that the j-th highest ability among the other $n-1$ participants does not exceed t; differentiating the binomial tail gives $Q_j'(t) = c_j\, t^{n-j-1}(1-t)^{j-1}$ with $c_j = \frac{(n-1)!}{(j-1)!\,(n-j-1)!}$, while $Q_n'(t) \equiv 0$ (equivalently, $c_n = 0$). Since $p_j(t) = Q_j(t) - Q_{j-1}(t)$, we have $p_j'(t) = Q_j'(t) - Q_{j-1}'(t)$, and Abel summation with $R_{N+1} = 0$ gives

$$\sum_{j=1}^{N} R_j\, p_j'(t) \;=\; \sum_{j=1}^{N} R_j\big[Q_j'(t) - Q_{j-1}'(t)\big] \;=\; \sum_{j=1}^{N} R_j Q_j'(t) - \sum_{j=0}^{N-1} R_{j+1} Q_j'(t) \;=\; \sum_{j=1}^{N} (R_j - R_{j+1})\, Q_j'(t),$$

where $Q_0'(t) \equiv 0$. Writing $d_j = R_j - R_{j+1} \geq 0$ and using $b(0) = 0$, we obtain

$$b^2(a) \;=\; \sum_{j=1}^{N} d_j\, v_j(a), \qquad v_j(a) \;=\; c_j \int_0^a t^{\,n-j}(1-t)^{j-1}\, dt \;\geq\; 0, \tag{3}$$

which is exactly Eq. (34) of the paper; the derivation of Eq. (34) itself is correct, and for $N = 1$ it reduces to $b^2(a) = (n-1)Ra^n/n$, i.e., Eq. (9) of the paper.

Step 2: Reduction to maximization over a simplex. Since the highest ability among n participants has density na^{n-1}, the sponsor's expected payoff is

$$\Pi \;=\; kn \int_0^1 \sqrt{\sum_{j=1}^{N} d_j\, v_j(a)}\; a^{\,n-1}\, da \;-\; (R + I). \tag{4}$$

The budget constraint $\sum_{j=1}^{N} R_j = R$ is equivalent to $\sum_{j=1}^{N} j\, d_j = R$. Define the simplex $\Delta = \{\hat{d} \in \mathbb{R}_+^n : \sum_{j=1}^{n} j\, \hat{d}_j = 1\}$ and

$$G(\hat{d}) \;=\; n \int_0^1 \sqrt{\sum_{j=1}^{n} \hat{d}_j\, v_j(a)}\; a^{\,n-1}\, da. \tag{5}$$

By homogeneity, any mechanism can be written as $d = R\hat{d}$ with $\hat{d} \in \Delta$, yielding $\Pi = k\sqrt{R}\, G(\hat{d}) - R - I$. Optimizing over R gives $R^* = \big(kG(\hat{d})/2\big)^2$ and $\Pi^*(\hat{d}) = \big(kG(\hat{d})\big)^2/4 - I$, which is strictly increasing in $G(\hat{d})$. Hence the sponsor's choice of award structure reduces to maximizing G over Δ.

Step 3: Concavity. For every $a \in [0,1]$, the map $\hat{d} \mapsto \sqrt{\sum_j \hat{d}_j v_j(a)}$ is concave, as a nondecreasing concave function of a linear map; integration against the positive measure $na^{n-1}da$ preserves concavity. Hence G is concave on the convex set Δ, and the Karush–Kuhn–Tucker (KKT) conditions are sufficient for global optimality.

Step 4: KKT verification at the single-prize point. The single-prize mechanism corresponds to the vertex $e_1 = (1, 0, \ldots, 0) \in \Delta$. It maximizes G on Δ if and only if

$$\frac{\partial G}{\partial \hat{d}_j}(e_1) \;\le\; j\,\frac{\partial G}{\partial \hat{d}_1}(e_1), \qquad \forall\, j = 2, \ldots, n. \tag{6}$$

At e_1, $\sum_j \hat{d}_j v_j(a) = v_1(a) = (n-1)a^n/n$, so

$$\frac{\partial G}{\partial \hat{d}_j}(e_1) \;=\; \frac{n}{2}\sqrt{\frac{n}{n-1}}\int_0^1 v_j(a)\, a^{\,n/2-1}\, da. \tag{7}$$

Substituting $v_j(a) = c_j \int_0^a t^{\,n-j}(1-t)^{j-1}\,dt$ and interchanging the order of integration (valid by Tonelli's theorem, since the integrand is nonnegative),

$$\int_0^1 a^{\,n/2-1}\int_0^a f(t)\,dt\,da \;=\; \int_0^1 f(t)\int_t^1 a^{\,n/2-1} da\,dt, \qquad \text{where} \quad f(t) \;=\; c_j\, t^{\,n-j}(1-t)^{j-1},$$

gives

$$\frac{\partial G}{\partial \hat{d}_j}(e_1) \;=\; \sqrt{\frac{n}{n-1}}\; c_j \left[B(n-j+1,\, j) \;-\; B\big(\tfrac{3n}{2} - j + 1,\, j\big)\right], \tag{8}$$

where $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ is the beta function. Since $B(x,y) = \frac{(x-1)!\,(y-1)!}{(x+y-1)!}$, we have the key identity

$$c_j\, B(n-j+1,\, j) \;=\; \frac{(n-1)!}{(j-1)!\,(n-j-1)!}\cdot\frac{(n-j)!\,(j-1)!}{n!} \;=\; \frac{n-j}{n}, \tag{9}$$

and for $j = 1$ this reduces to

$$\frac{\partial G}{\partial \hat{d}_1}(e_1) \;=\; \sqrt{\frac{n}{n-1}}\cdot\frac{n-1}{3n}.$$

Condition (6) is therefore equivalent to

$$c_j\, B\big(\tfrac{3n}{2} - j + 1,\, j\big) \;\ge\; \frac{n-j}{n} - \frac{j(n-1)}{3n} \;=\; \frac{3n - j(n+2)}{3n}. \tag{10}$$

For $j \ge 3$, the right-hand side of (10) satisfies $\frac{3n-j(n+2)}{3n} \le \frac{3n-3(n+2)}{3n} = -\frac{2}{n} < 0$, while the left-hand side is strictly positive; hence (10) holds strictly. For $j = 2$, $c_2 = (n-1)(n-2)$ and $B\big(\frac{3n}{2}-1,\, 2\big) = \int_0^1 t^{3n/2-2}(1-t)\,dt = \frac{4}{3n(3n-2)}$, so (10) reduces to $4(n-1)(n-2) \ge (n-4)(3n-2)$, i.e., $n(n+2) \ge 0$, which holds strictly (for $n = 3$ the right-hand side is negative). Condition (6) thus holds strictly for all $2 \le j \le n$.

Step 5: Uniqueness. For any $\hat{d} \in \Delta$ with $\hat{d} \ne e_1$, concavity of G, the constraint $\sum_j j\hat{d}_j = 1$ (i.e., $\hat{d}_1 - 1 = -\sum_{j\ge 2} j\hat{d}_j$), and the strictness of (6) imply

$$\begin{aligned} G(\hat{d}) \;&\le\; G(e_1) + \nabla G(e_1)^{\top}(\hat{d} - e_1) \\ &=\; G(e_1) + \sum_{j\ge 2}\Big[\frac{\partial G}{\partial \hat{d}_j}(e_1) - j\,\frac{\partial G}{\partial \hat{d}_1}(e_1)\Big]\hat{d}_j \;<\; G(e_1). \end{aligned} \tag{11}$$

Hence the single-prize mechanism is the unique global maximizer of G on Δ, and therefore uniquely maximizes the sponsor's expected payoff among all reasonable award mechanisms with the same total award. □

3 Implications

(i) Theorem 1 restores the conclusion of Proposition 7 in strengthened form; the original proof, Eqs. (A.20)–(A.26), is no longer needed. (ii) Corollary 2 of the paper—the dominance of the single-prize contest over the two-prize contest—follows as the special case $N = 2$. (iii) In the proportional intermediary fee scenario (Section 5 of the paper), the fee only rescales the effective award cost to $(1 + \rho)R$ and does not alter the award-structure problem; Steps 2–5 therefore apply verbatim, and Proposition 10 stands. (iv) All subsequent results built on the sponsor's single-prize choice—Propositions 8–12, Corollary 3, and Tables 2 and 3—together with the statements in the abstract, remain valid and unchanged. As a complement to the proof, we have also verified the conclusion numerically: for every $n = 3, \ldots, 1000$, the KKT ratio $\max_{j\geq 2} \left[(\partial G/\partial \hat{d}_j)/j\right]/(\partial G/\partial \hat{d}_1)$ is strictly below 1 (its maximum, 0.8327, occurs at $n = 1000$, $j = 2$), and direct global optimization over all award structures returns the single-prize mechanism in every instance examined.

The authors regret the error and apologize for any inconvenience it may have caused.

References

[1] Cheng, X., Gou, Q., Yue, J., & Zhang, Y. (2019). Equilibrium decisions for an innovation crowdsourcing platform. *Transportation Research Part E: Logistics and Transportation Review*, 125, 241–260.